Program: RFEM 5, RFEM 6, RSTAB 8, RSTAB 9, RF-FE-LTB, FE-LTB

Category: Second-Order Analysis, Isotropic Linear Elasticity, Warping, Member

Verification Example: 0050 – Cantilever Under Torsion Without Warping

0050 – Cantilever Under Torsion Without Warping

Description

A thin-walled cantilever of QRO-profile is fully fixed on the left end ($x = 0$), and the warping is enabled. The cantilever is subjected to a torque M at $x = L_1$ according to the **Figure 1** [1]. The problem is described by the following set of parameters.

Material	Steel	Modulus of Elasticity	E	210000.000	MPa
		Shear Modulus	G	81000.000	MPa
		Plastic Strength	f_y	355.000	MPa
Geometry	QRO Cantilever	Length	L	4.000	m
		Width and Height	b	200.000	mm
		Face Thickness	t	6.000	mm
Load		Torque	M	80.000	kNm
		Position	L_1	2.800	m

Small deformations are considered and the self-weight is neglected. Determine the maximum rotation $\varphi_{x,max}$ and control the values of the moments $M_{T,pri}$, $M_{T,sec}$ and M_ω at the point $x = L_1$.

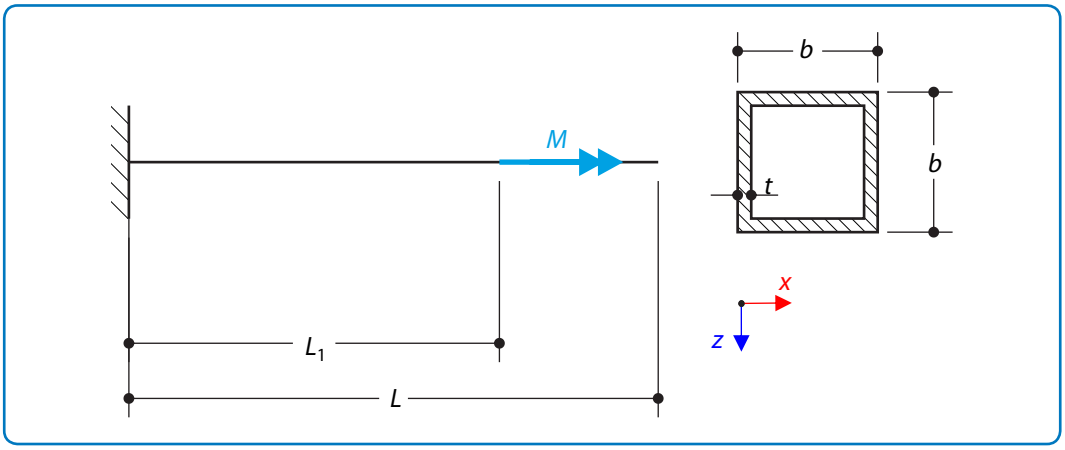

Figure 1: Problem sketch

Analytical Solution

The relative twist ϑ of the cantilever can be calculated according to the following formula

$$\vartheta = \frac{M}{GJ} = 2.252 \cdot 10^{-5} \text{ rad/mm} \qquad (50 - 1)$$

where J is the torsional constant[1]. For the given profile it is

$$J = 4.386 \cdot 10^7 \text{ mm}^4 \quad (50 - 2)$$

The maximum rotation about the x axis $\varphi_{x,\max}$ occurs at the point $x = L_1$ and it has the same value to the free end.

$$\varphi_{x,\max} = \vartheta L_1 = 0.063 \text{ rad} \quad (50 - 3)$$

For this thin-walled profile (radii in the corners are neglected) is the warping constant $C_\omega = 0$. Thus the moments M_{Tsec} and M_ω, which are defined below, should be zero on the full length of the profile. The primary torsional moment M_{Tpri} should be coincident with the total torsional moment M_T.

$$M_\omega(x) = -EC_\omega\varphi'(x) \quad (50 - 4)$$

$$M_{\text{Tsec}} = \frac{\text{d}M_\omega(x)}{\text{d}x} \quad (50 - 5)$$

RFEM and RSTAB Settings

- Modeled in RFEM 5.05.0029, RSTAB 8.05.0029 and RFEM 6.01, RSTAB 9.01
- The element size is $l_{FE} = 0.200$ m
- The number of increments is 5
- Isotropic linear elastic material model is used
- The structure is modeled using members
- Second-Order Analysis is used
- RF-FE-LTB and FE-LTB module is used in RFEM 5 and RSTAB 8
- Torsional Warping and Steel Design add-on is used in RFEM 6 and RSTAB 9

Results

Structure Files	Program
0050.01	RFEM 5 – RF-FE-LTB, RFEM 6
0050.02	RSTAB 8 – FE-LTB, RSTAB 9

	Analytical Solution	RFEM 5, RF-FE-LTB	Ratio	RSTAB 8, FE-LTB	Ratio
$\varphi_{x,\max}$ [rad]	0.063	0.063	1.000	0.063	1.000

[1] The torsional constant for the given profile can be approximately calculated as $J_{\text{approx.}} = t(b - t)^3$. The exact value is taken from RFEM / RSTAB.

	Analytical Solution	RFEM 6	Ratio	RSTAB 9	Ratio
$\varphi_{x,max}$ [rad]	0.063	0.063	1.000	0.063	1.000

In the following graphs there is demonstrated the behaviour of all the moments on the given cantilever calculated in RF-FE-LTB or FE-LTB module. The primary torsional moment M_{Tpri} should be coincident with the total torsional moment M_T. The moments M_{Tsec} and M_ω should be zero on the full length of the profile. It can be seen there is the affected area in the nearby of the loading point ($x = L_1$), when the warping is considered. Note that the above mentioned effect is getting smaller with the smoother mesh.

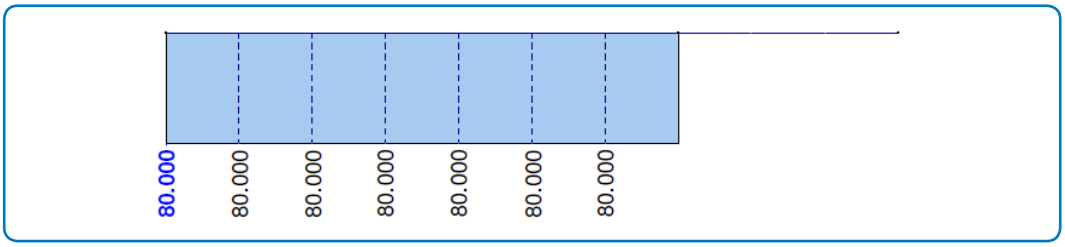

Figure 2: RFEM 5 – Total torsional moment M_T [kNm] behaviour

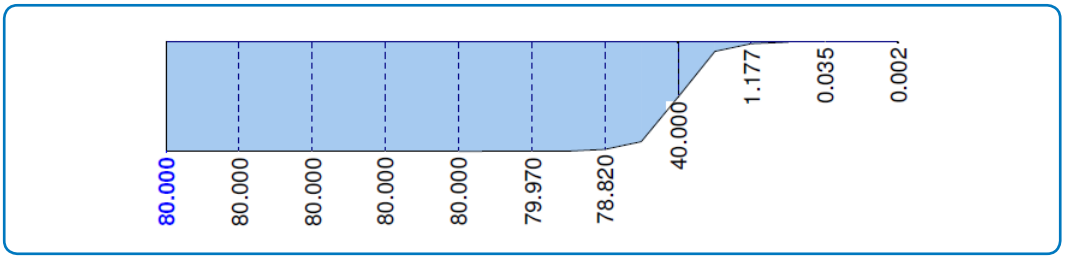

Figure 3: RFEM 5 – Primary torsional moment M_{Tpri} [kNm] behaviour

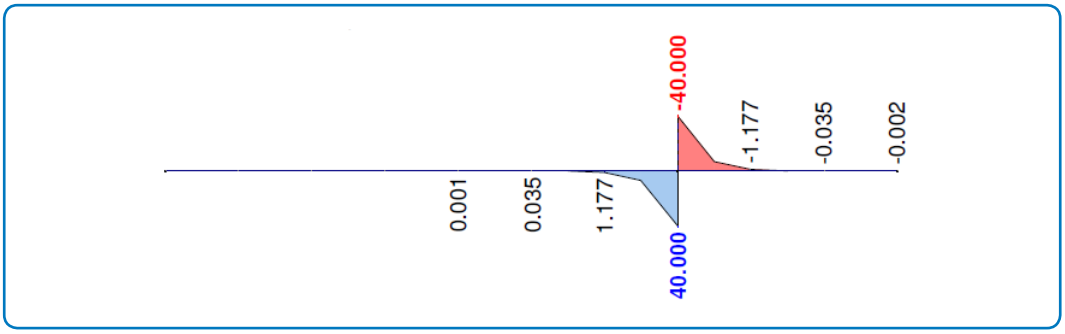

Figure 4: RFEM 5 – Secondary torsional moment M_{Tsec} [kNm] behaviour

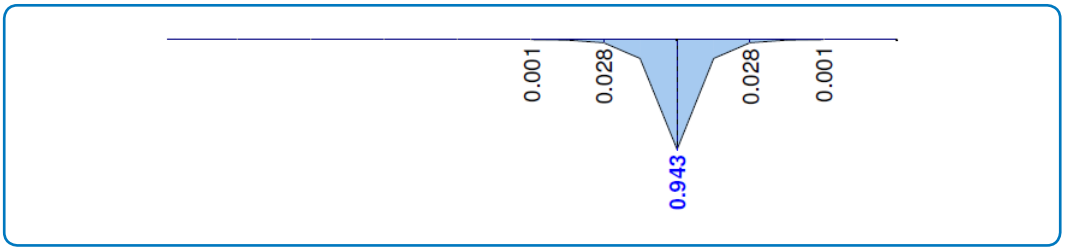

Figure 5: RFEM 5 – Warping moment M_ω [kNm^2] behaviour

The same task is also solved in RFEM 6 and RSTAB 9 with Torsional Warping Add-on. The results are shown in following graphs. It can be seen in RFEM 6 and RSTAB 9 results, there is no computational error due to the discretization.

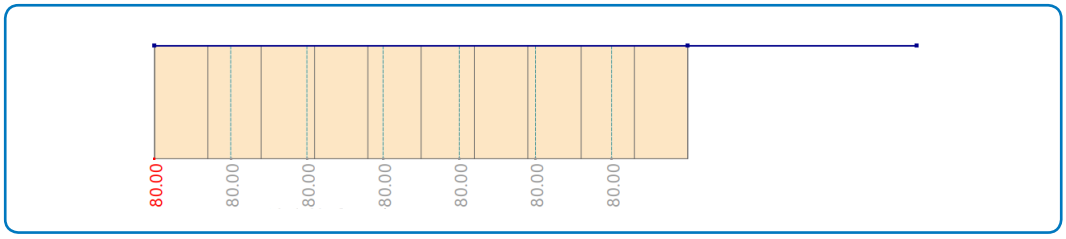

Figure 6: RFEM 6 – Total torsional moment M_T [kNm] behaviour

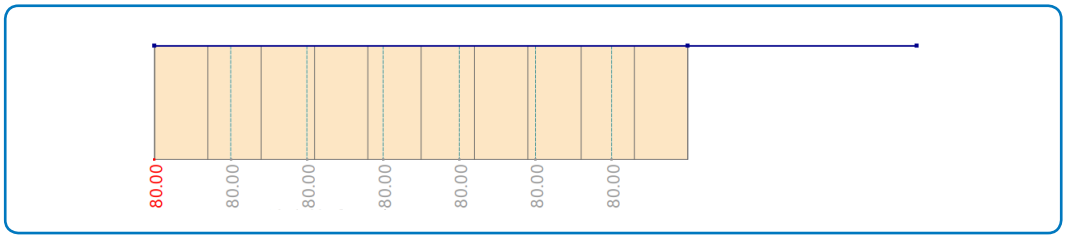

Figure 7: RFEM 6 – Primary torsional moment M_{Tpri} [kNm] behaviour

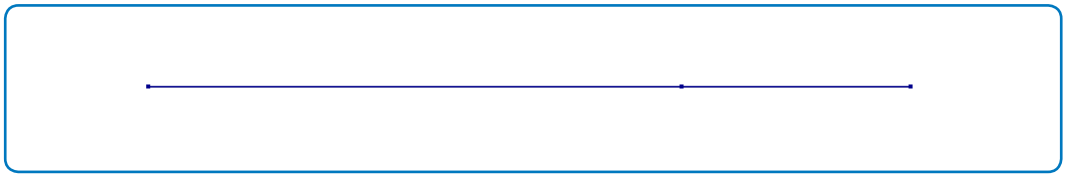

Figure 8: RFEM 6 – Secondary torsional moment M_{Tsec} [kNm] behaviour

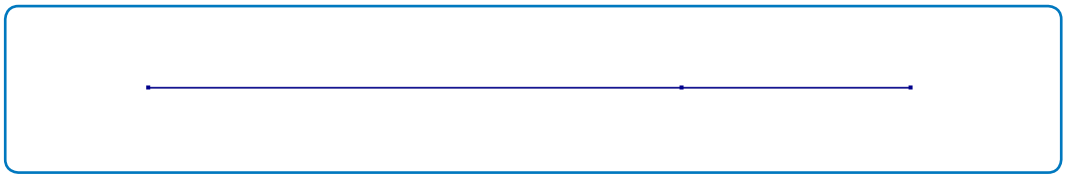

Figure 9: RFEM 6 – Warping moment M_ω [kNm^2] behaviour

References

[1] LUMPE, G. and GENSICHEN, V. *Evaluierung der linearen und nichtlinearen Stabstatik in Theorie und Software: Prüfbeispiele, Fehlerursachen, genaue Theorie*. Ernst, 2014.