0037 – Two-Dimensional Plasticity

Description

The wall is divided in the middle to the two parts. The upper and the lower part are made of an elastic-plastic and an elastic material respectively and both end planes are restricted to move in the vertical direction, see **Figure 1**. Due to the stability problems caused by the movements in horizontal direction the model is created as vertically symmetrical. Wall's self-weight is neglected, its edges are loaded with horizontal pressure p_h and the middle plane by vertical pressure p_v. Only small deformations are assumed. Determine the total deformation u at the test point A in the elastic part. In this example von Mises and Drucker-Prager plastic hypotheses as well as Tresca and Mohr-Coulomb plastic hypotheses will be considered. The difference between these models can be seen in **Figure 2**.

Material	Elastic-Plastic	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.000	–
		Plastic Strength	$f_{y,t} = f_{y,c}$	100.000	MPa
	Elastic	Modulus of Elasticity	E	210000.000	MPa
		Poisson's Ratio	ν	0.000	–
Geometry	Wall	Length	L	1.000	m
		Height	h	4.000	m
		Thickness	t	0.050	m
Load	Pressure	Horizontal	p_h	50.000	MPa
		Vertical	p_v	220.000	MPa

Analytical Solution

The total deformation u can be determined from the component deformations u_x and u_z.

$$u = \sqrt{u_x^2 + u_z^2} \tag{37 – 1}$$

The deformation components at the test point A can be calculated using the strains according to the Hook's law for the plane stress.

$$u_x = \varepsilon_x \frac{L}{2} = \frac{1}{E}\left(\sigma_x - \nu\sigma_z\right)\frac{L}{2} \tag{37 – 2}$$

$$u_z = \varepsilon_z \frac{h}{4} = \frac{1}{E}\left(\sigma_z - \nu\sigma_x\right)\frac{h}{4} \tag{37 – 3}$$

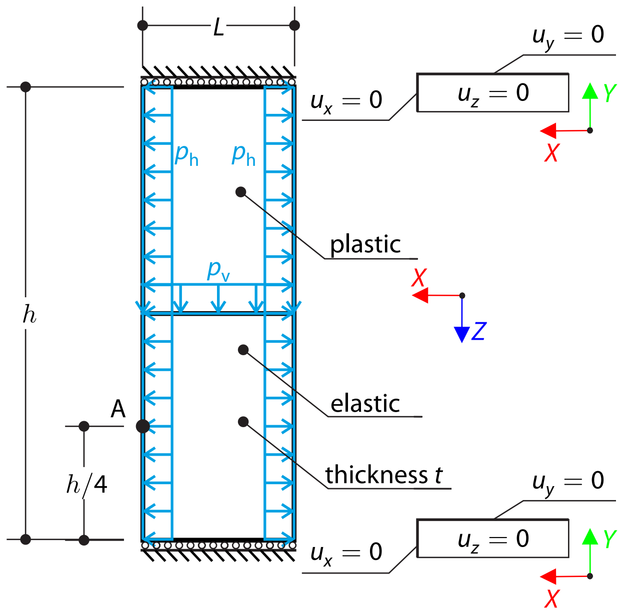

Figure 1: Problem sketch

In this case the Poisson's ratio is $\nu = 0$. According to von Mises and Drucker-Prager plastic hypotheses, a material behaves elastically if the following equation is satisfied:

$$\sigma_{\text{Mises}} = \sqrt{\sigma_x^2 + \sigma_z^2 - \sigma_x \sigma_z + 3\tau_{xz}} \leq f_{\text{y,t}} \tag{37 – 4}$$

where σ_x and σ_z are stresses in x and z directions respectively.

$$\sigma_x = p_{\text{h}} \tag{37 – 5}$$

$$\sigma_z = \frac{p_{\text{v}}}{2} \tag{37 – 6}$$

Using above mentioned equations the von Mises stress can be calculated.

$$\sigma_{\text{Mises}} = 95.394 \text{ MPa} \leq f_{\text{y,t}} \tag{37 – 7}$$

As can be seen, material behaves elastically and elastic stresses are equal to the loading stresses and the desired deformations can be calculated according to the above mentioned formulae.

$$u_x = 0.119 \text{ mm} \tag{37 – 8}$$

$$u_z = 0.524 \text{ mm} \tag{37 – 9}$$

$$u = 0.537 \text{ mm} \tag{37 – 10}$$

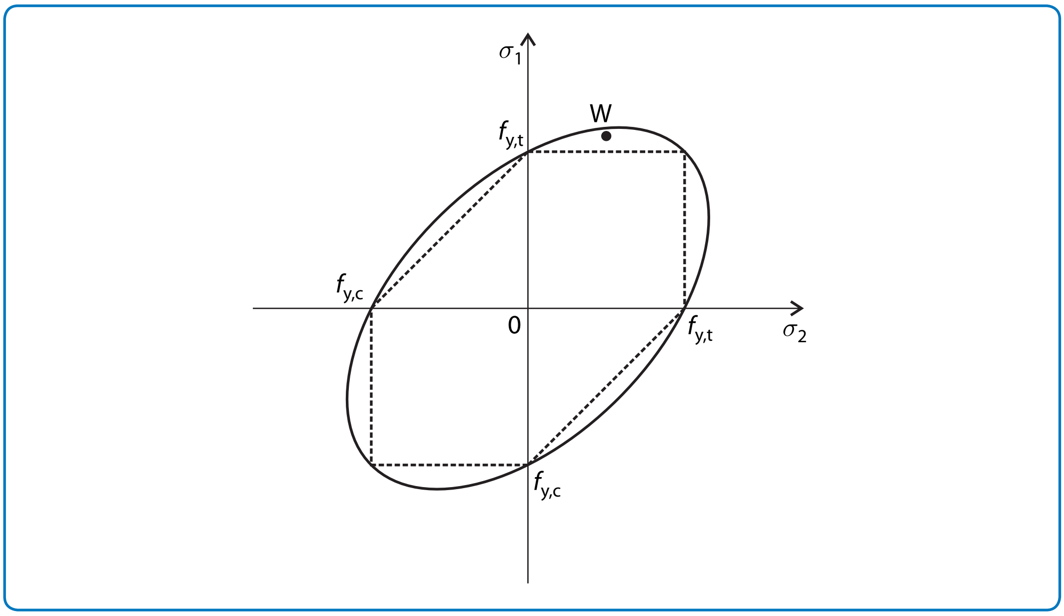

Figure 2: Von Mises and Drucker-Prager (solid line) and Tresca and Mohr-Coulomb (dashed line) plastic models in plane and considering equal plastic strength in tension and pressure. W is the working point of this verification example.

According to Tresca and Mohr-Coulomb plastic hypotheses, a material behaves elastically if the following equation is satisfied:

$$\sigma_{\text{Tresca}} = \max\left(\sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2}, \frac{|\sigma_x + \sigma_z + \sqrt{(\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2}|}{2}\right) \leq f_{\text{y,t}} \quad (37-11)$$

$$\sigma_{\text{Tresca}} = 110 \text{ MPa} \geq f_{\text{y,t}}$$

As can be seen, the equation **(37 – 11)** isn't satisfied and the material reached the plastic state. The stresses in the elastic part of the wall can be then expressed as follows:

$$\sigma_x = p_h \quad (37-12)$$

$$\sigma_z = p_v - f_{\text{y,t}} \quad (37-13)$$

And the deformations can be then obtained similarly to the previous case:

$$u_x = 0.119 \text{ mm} \quad (37-14)$$

$$u_z = 0.571 \text{ mm} \quad (37-15)$$

$$u = 0.584 \text{ mm} \quad (37-16)$$

RFEM Settings

- Modeled in version RFEM 5.26 and RFEM 6.01
- The element size is $l_{\text{FE}} = 0.050$ m

- Geometrically linear analysis is considered
- The number of increments is 10
- The Mindlin plate theory is used
- Nonsymmetric direct solver is used

Results

Structure File	Entity	Hypothesis
0037.01	Plate	von Mises
0037.02	Plate	Tresca
0037.03	Plate	Drucker-Prager
0037.04	Plate	Mohr-Coulomb
0037.05	Solid	von Mises
0037.06	Solid	Tresca
0037.07	Solid	Drucker-Prager
0037.08	Solid	Mohr-Coulomb

Hypothesis	Analytical Solution	RFEM 5 Plate		RFEM 5 Solid	
	u [mm]	u [mm]	Ratio [-]	u [mm]	Ratio [-]
von Mises	0.537	0.537	1.000	0.537	1.000
Tresca	0.584	0.583	0.998	0.583	0.998
Drucker - Prager	0.537	0.537	1.000	0.537	1.000
Mohr - Coulomb	0.584	0.583	0.998	0.583	0.998

Hypothesis	Analytical Solution	RFEM 6 Plate		RFEM 6 Solid	
	u [mm]	u [mm]	Ratio [-]	u [mm]	Ratio [-]
von Mises	0.537	0.537	1.000	0.537	1.000
Tresca	0.584	0.583	0.998	0.583	0.998
Drucker - Prager	0.537	0.537	1.000	0.537	1.000
Mohr - Coulomb	0.584	0.583	0.998	0.583	0.998